Secondary Stability of Ekman Layer Roll Vortices

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Roll Vortices in the Atmospheric Boundary Layer

Indirect (cloud streets) and direct (radar, lidar, SAR) observations

Cloud street (Brown, 1970)

SAR Image [Morrison et al., 2005]
- rolls induce momentum and heat fluxes
- can dominate fluxes induced by small-scale turbulence

Momentum Flux vs Shear in a Hurricane Boundary Layer

[Morrison et al., 2005]
The Inflection-Point Instability Scenario

Rolls are believed to emerge from inflection-point instability, reinforced by convective instability under convective conditions [Etling & Brown, 1993]

- Ekman spiral profile is linearly unstable [Lilly, 1966]
- 2D+1 equilibrium solutions obtained by weakly non-linear expansions [Foster, 1997 & 2005]
Near-Neutral / Shear-Driven PBL

- relevant for a range of situations, e.g. hurricane boundary layer
- has most important dynamical ingredients: shear, veering, lower boundary
- more tractable

- rolls obtained in 2D+1 neutral DNS at Re=150 but not Re=400 [Faller & Kaylor, 1966]
- rolls not observed in 3D neutral DNS at Re=400 [Coleman et al., 1990]
- LES require some buoyancy forcing to form rolls [Mason & Thompson, 1987; Moeng and Sullivan, 1994]
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Questions Addressed

- Does the flow reach another equilibrium, and how fast?
- Is that new equilibrium itself stable with respect to three-dimensional perturbations?
- Dependence on latitude? [Leibovich and Lele, 1985]
- Stable stratification?
Outline

1. Primary Instability
   - Model, Geometry, Scales, Parameters
   - Time Evolution toward Equilibrium
   - Rolls in Exact Equilibrium

2. Secondary Instability
   - Temporal and Spatial Scales
   - Structure and Energetics
   - Implications

3. Effects of Latitude and Stratification
   - Dependence on Latitude
   - Stable Stratification (work in progress)
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Model

Incompressible, rotating Navier-Stokes with constant turbulent viscosity

- Latitude $\lambda \Rightarrow$ Coriolis parameter, $f = 2\Omega_0 \sin \lambda \sim 10^{-4} \text{s}^{-1}$
- Ekman depth $\delta = \sqrt{2K/f} \sim 400 \text{ m}$
- Turbulent viscosity $K = f\delta^2/2 \sim 10 \text{ m}^2 \cdot \text{s}^{-1}$
- Geostrophic wind $U \sim 1 - 10 \text{ m} \cdot \text{s}^{-1}$
- Advective time scale $\tau_{ad} = \delta/U \sim 1 - 10 \text{ min}$
- Turbulent Reynolds number $Re = U\delta/K \simeq 200 - 500$
- Rossby number $Ro = U/f\delta = Re/2$

\[
\frac{\partial \mathbf{u}}{\partial t} = - \left( \omega + \frac{1}{Ro} (\mathbf{e}_z + \cotg \lambda \mathbf{e}_N) \right) \times \mathbf{u} - \nabla \left( P + \frac{\mathbf{u} \cdot \mathbf{u}}{2} \right) + \frac{1}{Re} \nabla^2 \mathbf{u}
\]
Linear instability

\[ u(x, z, t) = u_0(z) + u'_{2D}(x, z, t) \quad u'_{2D} \ll 1 \quad u'_{2D} = \tilde{u}(z)e^{\sigma t}e^{ik_1(x-\alpha t)} + c.c \]
Linear Instability (Lilly, 1966)

Ekman Spiral

growth rate vs wave vector $k_1$ at $Re = 500$
Linear Instability (Lilly, 1966)

Growth rate vs Reynolds number $Re$

Selected wave number vs $Re$

Selected wave number vs $Re$

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Time Evolution toward New Equilibrium?

\[ u(x, z, t) = u_0(z) + u'_{2D}(x, z, t) \quad \langle u'_{2D} \rangle_x \neq 0 \]

- 2D rolls obtained at Re=150 but not Re=400 [Faller & Kaylor, 1966]
- 3D rolls not observed at Re=400 [Coleman et al., 1990]

- Linear phase: constant phase velocity
- Followed by a strongly instationary evolution
- Then slow relaxation to equilibrated flow

Phase velocity vs time
Time Evolution toward New Equilibrium?

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Primary Instability
Secondary Instability
Effects of Latitude and Stratification
Summary

Rolls in Exact Equilibrium

\[ \mathbf{u}_1(x - ct, z) = \mathbf{u}_0(z) + \mathbf{u}'_{2D}(x - ct, z) \]

- axial velocity and vertical wind
- axial vorticity

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Secondary Stability of Ekman Layer Roll Vortices
Interim Summary

- fully non-linear, co-rotating, equilibrated rolls exist even at “high” Reynolds numbers
- are approached, but not always attained through temporal evolution of a small initial disturbance
- for practical purposes, the equilibrated rolls represent the flow adequately
- assumption of $y$--invariance is a strong restriction!
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- Time Evolution toward Equilibrium
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Secondary Instability
- Temporal and Spatial Scales
- Structure and Energetics
- Implications

Effects of Latitude and Stratification
- Dependence on Latitude
- Stable Stratification (work in progress)
Position of the Problem

\[ u(x + ct, y, z, t) = u_1(x, z) + u'_3D(x, y, z, t) \]

\[ u'_3D \ll 1 \quad u'_3D = e^{\sigma t} e^{i\gamma(y-c_2 t)} \tilde{u}(x, z) + c.c \]
Temporal and Spatial Scales

Growth rate vs axial wave number

Growth rate vs Reynolds Number

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Structure : Localization of Energy

\[ e(x, z, t) = \frac{1}{2} \left\langle u'^2 + v'^2 + w'^2 \right\rangle_y \]
Isosurface of total axial vorticity
\[ \partial_t \langle e(x, z, t) \rangle_{x,z} + D = \langle s_{AX} \rangle_{x,z} + \langle s_{CR} \rangle_{x,z} \]

\[ s_{AX} = -\langle v' u' \rangle_y \partial_x v_1 - \langle v' w' \rangle_y \partial_z v_1 \]
Kinetic Energy Budget

\[ \partial_t \langle e(x, z, t) \rangle_{x,z} + D = \langle s_{AX} \rangle_{x,z} + \langle s_{CR} \rangle_{x,z} \]

Energy production by cross–roll shear

\[ s_{CR} = - \langle u'w' \rangle_y (\partial_x w_1 + \partial_z u_1) - \langle u'u' \rangle_y \partial_x u_1 - \langle w'w' \rangle_y \partial_z w_1 \]
Implications

- Equilibrated rolls become unstable for $Re \geq 326$
- Growth rate of secondary instability is comparable to growth rate of primary instability; selected horizontal scale is about four times smaller
- Mechanism? (energetics can be misleading)
- Unstable mode localized in the updraft $\Rightarrow$ along-roll modulation of updrafts
- Nonlinear evolution of the unstable perturbation?

Cloud street (Brown, 1970)
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Why Latitude Matters

\[
\frac{\partial \mathbf{u}}{\partial t} = - \left( \omega + \frac{1}{Ro} \left( \mathbf{e}_z + \cotg \lambda \mathbf{e}_N \right) \right) \times \mathbf{u} - \nabla \left( P + \frac{\mathbf{u} \cdot \mathbf{u}}{2} \right) + \frac{1}{Re} \nabla^2 \mathbf{u}
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Primary Instability

Summary

Dependence on Latitude
Stable Stratification (work in progress)

Primary Instability [Leibovich & Lele, 1985]

Growth rate vs Reynolds Number

Selected wave vector vs Re

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Secondary Stability of Ekman Layer Roll Vortices
Dependence of Secondary Instability on Latitude

Growth rate $\sigma$ vs Reynolds Number

$\sigma$ vs Re

Selected axial wave vector $k_2$ vs Re

$\sigma_2$ vs Re

$k_2$ vs Re

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Secondary Stability of Ekman Layer Roll Vortices
Richardson Number

- Buoyancy $g(\theta - \theta_0)/\theta_0 = N^2 \hat{z} + \hat{b}$
- Richardson Number $Ri = (N\delta/U)^2$
- Turbulent diffusivity
- Turbulent Prandtl number $Pr = K/K_b$
- Adimensionalize $b = \hat{b}/NU$
- Boussinesq equations

\[
\frac{\partial \mathbf{u}}{\partial t} + \left( \omega + \frac{1}{Ro} \mathbf{e}_z \right) \times \mathbf{u} + \nabla \left( P + \frac{\mathbf{u} \cdot \mathbf{u}}{2} \right) = Ri^{1/2} b \mathbf{e}_z + \frac{1}{Re} \nabla^2 \mathbf{u}
\]

\[
\frac{\partial b}{\partial t} + \nabla b \cdot \mathbf{u} = -Ri^{1/2} w + \frac{1}{Pr \, Re} \nabla^2 b
\]

Mechanical energy $\frac{u^2 + v^2 + w^2 + b^2}{2}$
Effect on Equilibrated Rolls

Roll energy vs Richardson number

- Potential energy
- Kinetic energy
- Total

$N^2_{\text{min}}$ vs $Ri$

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Effect on secondary instability: overturning?

Reminiscent of PhD work by J. Barnard on the strongly stratified Ekman layer.
Summary

- Fully non-linear, co-rotating, equilibrated rolls are approached, but not always attained through temporal evolution of a small initial disturbance.
- Growth rate of secondary instability is comparable to growth rate of primary instability; scale is about four times smaller.
- And quite sensitive to the horizontal Coriolis component.

- Nonlinear evolution of the unstable secondary perturbation?
- Effect of stratification? Might in fact enhance the secondary instability?
- Strong stratification? Turbulent bursts initiated by transient growth of perturbations?
Summary

- **fully non-linear, co-rotating, equilibrated rolls are approached, but not always attained** through temporal evolution of a small initial disturbance.

- **growth rate** of secondary instability is **comparable** to growth rate of primary instability; **scale** is about **four times smaller**.

- And quite **sensitive** to the horizontal Coriolis component.

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